Example :- Show that for a cylindrical resister of length l, radius r and resistivity ρ, the rate of flow of energy P, at which energy flows into the resister through its cylindrical surface (calculated by integrated the Poynting vector over this surface) is equal to the rate at which Joule heat is produced

i.e.

$$P = I^2 R$$

Solution :- As in case of conductor

$$J = \sigma E \quad \text{i.e. } E = \rho J$$

The electric field is parallel to the direction of J. In addition to the above electric field whose direction is given by 'right hand thumb' rule and magnitude by Ampares Law i.e.

$$\oint_C H \cdot dl = \int_S J \cdot ds$$

i.e. $$H \, 2\pi r = J \pi r^2$$

i.e. $$H = \frac{1}{2} J r$$

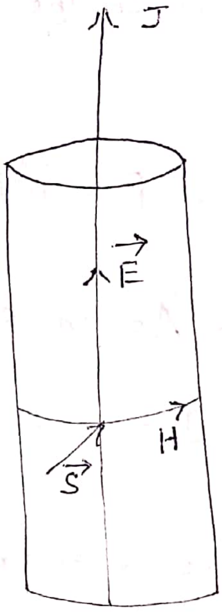
The direction of E and H are perpendicular to each other as shown in fig. So the Poynting Vector $S = E \times H$ will be perpendicular to the plane of E and H and into the paper.

Its magnitude will be

$$S = EH = \rho J \times \frac{1}{2} J r$$

i.e. $$S = \frac{1}{2} \rho J^2 r$$

As a consequence of this ~~plate~~ rate of flow of field energy into the resister through its cylindrical surface will be



$$P = -\oint_S S \cdot ds$$

$$= \oint_S S ds$$

[as S and ds are anti parallel]

i.e. $\int \frac{1}{2} \rho J^2 r ds = \frac{1}{2} \rho J^2 r \times 2\pi r l$ $\left[\text{as} \int ds = 2\pi r l\right]$

i.e. $P = \rho J^2 (\pi r^2 l) = I^2 \rho \frac{l}{\pi a^2}$ $\left[\text{as } J = \frac{I}{\pi r^2}\right]$

i.e. $P = I^2 R$ $\left[\text{as } R = \rho \frac{l}{\pi r^2}\right]$

Example :- A parallel plate capacitor as shown in fig. is being charged. Show that the rate at which energy flow into the capacitor from the surrounding space. (Calculated by integrating the Poynting Vector over the cylindrical boundary of the volume) is equal to the rate at which the stored electric energy increases i.e.

$$P = \pi r^2 h \epsilon . E \frac{\partial E}{\partial t}$$

Solution:— Let E be the electric field at any instant when the capacitor is being charged. Naturally E will be perpendicular to the plates and will be directed from the positive to the negative plate.

Now as a changing electric field produces a magnetic field given by

$$\oint B . dl = \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

ie
$$\oint H . dl = \frac{\partial}{\partial t} \int \epsilon_0 E . ds$$

$$\left[\text{as } B = \mu_0 H \text{ and } \phi_E = \int E . ds \right]$$

or
$$H (2\pi r) = \frac{\partial}{\partial t} (\epsilon_0 E) \pi r^2$$

or
$$H = \frac{1}{2} \epsilon_0 r \frac{\partial E}{\partial t}$$

The direction of H is determind by right hand thumb rule and is shown in fig. The direction of E and H are perpendicular to each other, so the Poyinting Vector $S = E \times H$

will be perpendicular to the plane of E and H and into paper. Its magnitude will be.

$$S = EH \sin 90^\circ$$
$$= \frac{1}{2} \epsilon_0 r E \frac{\partial E}{\partial t}$$

As a concequence of this the rate of flow of field energy into the capacitor through its cylindrical surface will be

$$P = -\oint_S S \cdot ds = \oint_S S\, ds$$

(as S and ds are antiparallel)

i.e. $$P = \int \frac{1}{2} \epsilon_0 r E \frac{\partial E}{\partial t}\, ds$$
$$= \frac{1}{2} \epsilon_0 r E \frac{\partial E}{\partial t}\, 2\pi r h$$

i.e. $$P = \pi r^2 h \epsilon_0 E \frac{\partial E}{\partial t}$$

This is the required result.

The example is

$$U = \pi r^2 h \left(\frac{1}{2} \epsilon_0 E^2\right)$$

i.e. $$\frac{\partial U}{\partial t} = \pi r^2 h \frac{\partial}{\partial t}\left(\frac{1}{2} \epsilon_0 E^2\right)$$
$$= \pi r^2 h \epsilon_0 E \frac{\partial E}{\partial t}$$

Continue